Environmental CPT Violation in an Expanding Universe in String Theory

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We consider a model of an expanding Universe in string theory that yields 'environmental' CPT violation for fermions, in the sense of different dispersion relations for fermions and antifermions. These are induced by a cosmological background with constant torsion provided by the Kalb-Ramond antisymmetric tensor field (axion) of the string gravitational multiplet. This effect induces different densities of neutrinos and antineutrinos while in chemical equilibrium, offering new scenarios for leptogenesis and baryogenesis even in the absence of CP violation.

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I. INTRODUCTION AND SUMMARY

The visible matter in the Universe is composed overwhelmingly of matter particles, with no detectable concentrations of anti-matter particles. However, according to the standard Big Bang theory in the absence of a primordial matter-antimatter asymmetry, matter and antimatter would have been present in equal amounts in the early radiation-dominated Universe. The observation of charge-parity (CP) violation in particle physics [1] prompted Andrei Sakharov [2] to conjecture that fundamental particle interactions could have generated a Baryon Asymmetry in the Universe (BAU), if the following conditions could all be met:

- Baryon-number-violating interactions that allow the generation of states with $B \neq 0$ starting from an initial state with $B = 0$, where B is the baryon number;

- Interactions capable of distinguishing between matter and antimatter - assuming CPT symmetry, this would require violation of both C and CP;

- Since matter-antimatter asymmetry is impossible in chemical equilibrium, one also requires some breakdown of chemical equilibrium during an epoch in the early Universe.

The asymmetry between the baryon (n_B) and anti-baryon ($n_{\overline{B}}$) number densities observed today is estimated to require [3]

$$\Delta n = \frac{n_B - n_{\overline{B}}}{n_B + n_{\overline{B}}} \sim \frac{n_B - n_{\overline{B}}}{s} = (8.4 - 8.9) \times 10^{-11} \tag{1}$$

at the early stages of the expansion for times $t \sim 10^{-6}$ s and temperatures $T \sim 1$ GeV, where s denotes the entropy density. The CP violation observed [1] and predicted within the Standard Model (SM) of particle physics could not have induced an asymmetry as large as (1) [4]. There are several ideas that go beyond the SM, e.g., grand unified theories (GUTs), supersymmetry, extra dimensions and the decays of massive sterile neutrinos, that provide additional sources of CP violation and hence scenarios for generating the BAU: see [5–11] for recent examples.

A basic assumption in the Sakharov scenario is that CPT symmetry [12] (where T denotes the time-reversal operation) holds in the very early Universe. Indeed, CPT invariance is intrinsic to all known *local* effective *relativistic* field theories without gravity, such as those upon which current particle-physics phenomenology is based. However, CPT may be violated, e.g., as the result of a breakdown of Lorentz symmetry [13] or of quantum coherence, such as might occur in some theories of quantum gravity [14, 15], and the requirement of a breakdown of equilibrium can be dropped if the requirement of CPT is relaxed [16]. One attractive scenario is that the violation of CPT (denoted by CPTV) early in the expansion of the Universe, might have generated a primordial lepton asymmetry, after which either sphaleron processes [17] or Baryon-Lepton (B-L) conserving processes in some GUT communicated this lepton asymmetry to the baryon sector; in such a scenario, this mechanism which we call CPTV leptogenesis, produces the observed BAU. CPTV in the early Universe would thus have obviated the need for extra sources of CP violation, such as sterile neutrinos and/or supersymmetry, in order to obtain the observed BAU.

In this paper we shall present a new model for CPTV leptogenesis, proposed in a preliminary form in [18]. Our approach follows broadly an earlier framework proposed in [19, 20], but differs crucially in that the full gravitational multiplet [21] arising in string theory is used. The model is based on a variant of a stringy theory prototype for an expanding Universe proposed in [22], which is extended appropriately to consider the effects on fermion propagation in a background with constant non-zero torsion provided by the antisymmetric Kalb-Ramond tensor in the gravitational multiplet of the string. The leptogenesis in this model is due to torsion-induced CPTV in the dispersion relations of fermions and antifermions. Although the model presented here is a toy prototype, it contains some nontrivial features that may survive the extension to more realistic and detailed string cosmologies.

The structure of this paper is as follows: in Section II we discuss the string model of [22] and the underlying world-sheet conformal field theory formalism that leads to a non-trivial torsion background, and we explain how such a background leads to CPTV dispersion relations for fermions and antifermions. We then discuss how this would lead to different number densities for neutrinos and antineutrinos, providing a possible scenario for leptogenesis without the need for any extra sources of CP violation. Finally, an outlook is presented in Section III.

II. ENVIRONMENTAL CPT VIOLATION IN A STRING COSMOLOGICAL BACKGROUND GEOMETRY WITH TORSION

The massless gravitational multiplet in string theory contains the graviton, described by a spin-2 symmetric tensor $g_{\mu\nu}$, a spin-0 scalar dilaton, Φ, and an antisymmetric tensor, $B_{\mu\nu}$. This Kalb-Ramond field appears in the string

effective action only through its totally-antisymmetric field strength, $H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$, where $[\dots]$ denotes antisymmetrization of the indices within the brackets. The calculation of string amplitudes [23] shows that $H_{\mu\nu\rho}$ plays the role of *torsion* in a generalized connection. In the Einstein frame, to $O(\alpha')$ the four-dimensional bosonic part of the effective action of the string is:

$$S = \frac{M_s^2 V^c}{16\pi} \int d^4x \sqrt{-g}\left(R(g) - 2\partial^\mu\Phi\partial_\mu\Phi - \frac{1}{12}e^{-4\Phi}H_{\mu\nu\rho}H^{\mu\nu\rho} + \dots\right), \tag{2}$$

where $M_s = 1/\sqrt{\alpha'}$ is the string mass scale, the compactification volume $V^{(c)}$ and the compact radii are expressed dimensionlessly in units of $\sqrt{\alpha'}$. It can be shown that the term in S that is the square of the field strength can be combined with the Einstein scalar curvature term $R(g)$ to form a generalized curvature term $\overline{R}(g, \overline{\Gamma})$. This generaliszed curvature is defined in terms of a generalized Christoffel symbol (connection) $\overline{\Gamma}$:

$$\overline{\Gamma}^\lambda_{\ \mu\nu} = \Gamma^\lambda_{\ \mu\nu} + e^{-2\Phi}H^\lambda_{\mu\nu} \equiv \Gamma^\lambda_{\ \mu\nu} + T^\lambda_{\ \mu\nu}, \tag{3}$$

where $\Gamma^\lambda_{\ \mu\nu} = \Gamma^\lambda_{\ \nu\mu}$ is the torsion-free Einstein-metric connection, and $T^\lambda_{\ \mu\nu} = -T^\lambda_{\ \nu\mu}$ is the torsion.

By general covariance, the four-dimensional Lagrangian $\mathcal{L}_f$ for a spin-1/2 Dirac fermion ψ in this background with torsion is (up to constants of proportionality):

$$\mathcal{L}_f \sim \sqrt{-g}\left(i\overline{\psi}\,\gamma^a D_a\,\psi - m\overline{\psi}\,\psi\right): \quad D_a = \partial_a - \frac{i}{4}\overline{\omega}_{bca}\,\sigma^{bc}, \quad \sigma^{ab} = \frac{i}{2}[\gamma^a, \gamma^b], \tag{4}$$

where Latin indices refer to tangent space, and ω, the generalized spin connection (with torsion), is:

$$\overline{\omega}_{bcd} = e_{b\lambda}\left(\partial_a e_a^\lambda + \overline{\Gamma}^\lambda_{\ \gamma\mu} e_c^\gamma e_a^\mu\right). \tag{5}$$

Using properties of the Dirac γ-matrices, the Lagrangian (4) can be cast in the form

$$\mathcal{L} = \sqrt{-g}\,\overline{\psi}\left(i\gamma^a\partial_a - m + \gamma^a\gamma^5 B_a\right)\psi, \quad B^d = \epsilon^{abcd}e_{b\lambda}\left(\partial_a e_c^\lambda + \Gamma^\lambda_{\ \nu\mu}e_c^\nu e_a^\mu\right). \tag{6}$$

The space-time curvature background therefore has the effect of inducing an 'axial' background field B_a that is known to be non-trivial in certain anisotropic space-time geometries, such as Bianchi-type cosmologies or in regions of space-time near rotating (Kerr) primordial black holes [20, 24–26].

For the application to particle-antiparticle asymmetry, we are interested in an axial field B_a that is constant in some local frame, in which case CPT is violated 'environmentally' as a consequence of background-induced Lorentz violation, since the dispersion relations of fermions and antifermions differ in such backgrounds. Explicitly we have [20, 24–26].

$$E = \sqrt{(\vec{p} - \vec{B})^2 + m^2} + B_0, \quad \overline{E} = \sqrt{(\vec{p} + \vec{B})^2 + m^2} - B_0. \tag{7}$$

Similar considerations apply to (left-handed) Majorana fermions, in which case there is a difference between the dispersion relations for left-handed spinors and their conjugate (right-handed) spinors. In this case, the coupling of the (left-handed) Majorana fermions to the background four-vector B^a has the following form in the Weyl representation [24–26].

$$\mathcal{L}_{\text{major}} \ni \sqrt{-g}\left(\overline{\psi}_L^c\gamma^a\psi_L^c - \overline{\psi}_L\gamma^a\psi_L\right)B_a, \tag{8}$$

which implies different (CPTV) dispersion relations of the form (7) for (left-handed) Majorana spinors and their (right-handed) conjugates.

The existence of a frame in which the four-vector B^a is constant in space-time has not been demonstrated in the literature existing so far. However, as argued in [18] and shown below, this is the case in the string background we consider here, in which the torsion provides a space-time-independent B^0 and vanishing $\vec{B}$. The torsion $T_{\mu\nu\rho}$ associated with the generalized connection $\overline{\Gamma}^\rho_{\mu\nu}$ (3) yields *non-zero* contributions to the fermion interactions in (6), with $e_{b\lambda}\overline{\Gamma}^\lambda_{\ \gamma\mu}e_c^\gamma e_a^\mu$. Even in the case of constant vierbeins, i.e., in the limit of flat Minkowski space-time, there are non-trivial interactions between the fermions and the torsion part of the spin-connection.

Solutions to the conformal invariance conditions of the low-energy string effective action that are exact to all orders in α' have been presented in [22]. The antisymmetric tensor field strength in the four 'large' (uncompactified) dimensions of the string can be written uniquely as

$$H_{\mu\nu\rho} = e^{2\Phi}\epsilon_{\mu\nu\rho\sigma}\partial^\sigma b(x), \tag{9}$$

where $\epsilon_{0123} = \sqrt{g}$ and $\epsilon^{\mu\nu\rho\sigma} = |g|^{-1}\epsilon_{\mu\nu\rho\sigma}$, with g the metric determinant. The field $b(x)$ is a pseudoscalar axion-like field. The dilaton Φ and axion b fields appear as Goldstone bosons of spontaneously broken scale symmetries of the string vacuum, and so are exactly massless classically. Such fields appear in the effective action only through their derivatives. In the exact solution of [22] in the string frame both the dilaton and axion fields are linear in the target time X^0 in the σ-model (Jordan) frame: $\Phi(X^0), b(X^0) \sim X^0$. This solution shifts the minima of all the fields in the effective action that couple to the dilaton and axion by an amount that is independent of space and time.

In the Einstein frame that is relevant for cosmological observations, the temporal component of the metric is normalized to $g_{00} = +1$ by an appropriate change of the time coordinate. In this setting, the solution of [22] leads to a Friedmann-Robertson-Walker (FRW) metric with scale factor $a(t) \sim t$, where t is the FRW cosmic time and the dilaton field Φ behaves as

$$\Phi(t) = -\ln t + \phi_0 \ , \tag{10}$$

where ϕ_0 is a constant, and the axion field $b(x)$ is linear in t, cf, Eq. (12) below. There is an underlying world-sheet conformal field theory with central charge

$$c = 4 - 12Q^2 - \frac{6}{n+2} + c_I \ , \tag{11}$$

where $Q^2(> 0)$ is the central-charge deficit and c_I is the central charge associated with the world-sheet conformal field theory of the compact 'internal' dimensions of the string model [22]. The world-sheet ghosts must cancel because of the requirement of reparametrization invariance of the world-sheet co-ordinates entails that $c = 26$.

The solution for the axion field is

$$b(x) = \sqrt{2}e^{-\phi_0}\sqrt{Q^2}\frac{M_s}{\sqrt{n}}t \ , \tag{12}$$

where M_s is the string mass scale and n is a positive integer, associated with the level of the Kac-Moody algebra of the underlying world-sheet conformal field theory. For non-zero Q^2 there is an additional dark energy term in the effective target space-time action of the string [22] of the form $\int d^4x\sqrt{-g}e^{2\Phi}(-4Q^2)/\alpha'$. The linear axion field (12) remains a non-trivial solution even in the limit of a static space-time with a constant dilaton field [22]. In this case the space-time is an Einstein universe with a positive cosmological constant and constant positive curvature proportional to $6/(n+2)$. For the solutions of [22], the covariant torsion tensor $e^{-2\Phi}H_{\mu\nu\rho}$ is constant, as can be seen from (3) and (9, since the exponential dilaton factors cancel out in the relevant expressions. Only the spatial components of the torsion are nonzero in this case: $T_{ijk} \sim \epsilon_{ijk}\dot{b} = \epsilon_{ijk}\sqrt{2Q^2}e^{-\phi_0}\frac{M_s}{\sqrt{n}}$, where the dot denotes a derivative with respect to t.

On the basis of (3), (9) and (6), we observe that only the temporal component B^0 of the vector B^d is non-zero and constant, of order[1]

$$B^0 = \epsilon^{ijk}T_{ijk} = 6\sqrt{2Q^2}e^{-\phi_0}\frac{M_s}{\sqrt{n}} \text{ GeV} > 0 \,. \tag{13}$$

One obtains from the dispersion relations (7) different populations of fermions and antifermions while in thermal equilibrium, as a result of the following expressions for the particle and antiparticle distribution functions:

$$f(E,\mu) = [\exp(E(\vec{p}) - \mu)/T) \pm 1]^{-1} \ , \quad f(\overline{E(\vec{p})},\bar{\mu}) = [\exp(\bar{E}(\vec{p}) - \bar{\mu})/T) \pm 1]^{-1} \ , \tag{14}$$

where $\vec{p}$ is the $3-$momentum and μ the chemical potential. Our convention is that an overline over a quantity refers to an antiparticle, the $+$ sign applies for a fermionic (anti-)particle and the $-$ applies for a bosonic (anti-)particle [2].

Since the torsion couples universally to all fermonic species, the above considerations apply to all flavours of fermions, including both quarks and leptons. In the following, we first concentrate on the case of light (left-handed) neutrinos, with a view to their potential importance in leptogenesis. In this case, we postulate an early epoch during which lepton-number-violating processes are sufficiently rapid to maintain chemical equilibrium for neutrinos and antineutrinos. As we see from Eqs. (13), (7) and (14), one obtains when $B^0 \ll T$ [18, 20, 24–26]:

$$\Delta n_\nu \equiv n_\nu - n_{\overline{\nu}} \sim g^\star T^3\left(\frac{B_0}{T}\right) \tag{15}$$

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[1] We note that the torsion-free gravitational part of the connection (for the FRW or flat case) yields a vanishing contribution to B^0.

[2] We note that in the discussion of [22], it was observed that the central charge deficit $Q^2 > 0$ implies tachyonic (i.e. negative) shifts in the mass squared of the boson fields, while fermion masses were not affected. We see in this work that the presence of antisymmetric torsion does affect the dispersion relations (7) for fermions .

where $g^\star$ is the number of degrees of freedom for the (relativistic) neutrino, and an excess of particles over antiparticles is predicted when $B_0 > 0$. If B^0 is independent of temperature, as assumed but not demonstrated in the models of [20, 24–26], the CPTV asymmetry induced by the background decreases with the temperature.

At temperatures $T < T_d$, where T_d denotes the temperature at which lepton-number violating processes decouple, which depends on the details of the underlying model, the ratio of the net lepton number ΔL (neutrino asymmetry) to entropy density (which scales as T^3) remains constant,

$$\Delta L(T < T_d) = \frac{\Delta n_\nu}{s} \sim \frac{B_0}{T_d} \,, \tag{16}$$

As in other leptogenesis scenarios, this lepton asymmetry can then be communicated to the baryon sector to produce the observed baryon asymmetry via the B-L-conserving sphaleron processes of the standard model or B-L-conserving processes in the context of some grand unified theory. It should be emphasized, however, that unlike conventional leptogenesis scenarios there is no need to postulate any CP violation in any lepton-number-violating processes. The sign of B_0 determines whether there is an abundance of neutrinos over antineutrinos or vice versa. For successful baryogenesis in a B-L conserving scenario, e.g., exploiting sphalerons in the Standard Model, we need $B_0 > 0$.

We consider first the case of massive Majorana neutrinos, in which case torsion induces oscillations between neutrinos and antineutrinos, as in the original suggestion of Pontecorvo [27, 28]. These oscillations are induced by the mixing of neutrino and antineutrino states to produce mass eigenstates due to the constant 'environmental' field B^0 [25, 26]. To see this, we consider the Lagrangian for Majorana neutrinos in the presence of B_a, written in terms of two-component (Weyl) spinors ψ, ψ^c (a generic four-component Majorana spinor Ψ may be written in our notation as $\Psi = \begin{pmatrix} \psi_L^c \\ \psi_L \end{pmatrix}$, where from now on we omit the left-handed suffix L):

$$\mathcal{L}_\nu = \sqrt{-g} \Big[\begin{pmatrix} \psi^{c\dagger} & \psi^\dagger \end{pmatrix} \frac{i}{2} \gamma^0 \gamma^\mu D_\mu \begin{pmatrix} \psi^c \\ \psi \end{pmatrix} - \begin{pmatrix} \psi^{c\dagger} & \psi^\dagger \end{pmatrix} \begin{pmatrix} -B_0 & -m \\ -m & B_0 \end{pmatrix} \begin{pmatrix} \psi^c \\ \psi \end{pmatrix} \,, \tag{17}$$

where we assume for now that the neutrino has only lepton-number-violating Majorana-type masses [3]. We see from (17) that, in the presence of torsion, there are non-trivial and unequal diagonal lepton-number-conserving entries in the mass matrix $\mathcal{M}$ for ψ and ψ^c:

$$\mathcal{M} = \begin{pmatrix} -B_0 & -m \\ -m & B_0 \end{pmatrix} \,. \tag{18}$$

The mass matrix (18) is hermitean, so can be diagonalised by a unitary matrix, leading to two-component mass eigenstates $|\chi_{i,j}\rangle$ that are mixtures of the energy eigenstates $|\psi\rangle$ and $|\psi^c\rangle$:

$$|\chi_1\rangle = \mathcal{N}^{-1} \left\{ \left(B_0 + \sqrt{B_0^2 + m^2} \right) |\psi^c\rangle + m |\psi\rangle \right\} \,,$$
$$|\chi_2\rangle = \mathcal{N}^{-1} \left\{ -m |\psi^c\rangle + \left(B_0 + \sqrt{B_0^2 + m^2} \right) |\psi\rangle \right\} \,, \tag{19}$$

where

$$\mathcal{N} \equiv \left[2 \left(B_0^2 + m^2 + B_0 \sqrt{B_0^2 + m^2} \right) \right]^{1/2} \,, \tag{20}$$

with mass eigenvalues

$$m_{1,2} = \mp \sqrt{B_0^2 + m^2} \,. \tag{21}$$

Notice that in the presence of B_0 the mass eigenstates are *different* from the energy eigenstates of the Hamiltonian.

The above mixing can be expressed by writing the four-component neutrino spinor in terms of ψ and ψ^c using an angle θ [26]:

$$\nu \equiv \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \psi^c \\ \psi \end{pmatrix} \quad : \quad \tan\theta \equiv \frac{m}{B_0 + \sqrt{B_0^2 + m^2}} \,. \tag{22}$$

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[3] The extension to Dirac-type masses involving sterile neutrino will be discussed later on.

It is readily seen that the four-component spinor ν is also Majorana, as it satisfies the Majorana condition $\nu^c = \nu$. We note that in the absence of torsion, $B_0 \to 0$, the mixing angle between the two-component spinors ψ and ψ^c is maximal: $\theta = \pi/4$, whereas it is non-maximal when $B_0 \neq 0$.

The mixing (22) enables us to understand the difference between the densities of fermions and antifermions mentioned earlier (15). The expectation values of the number operators of $\chi_i, i = 1, 2$ in energy eigenstates are given by:

$$N_{\chi_1} = <: \chi_1^\dagger \chi_1 :> = \cos^2\theta <: \psi^{c\dagger} \psi^c :> + \sin^2\theta <: \psi^\dagger \psi :> \ ,$$
$$N_{\chi_2} = <: \chi_2^\dagger \chi_2 :> = \sin^2\theta <: \psi^{c\dagger} \psi^c :> + \cos^2\theta <: \psi^\dagger \psi :> \ , \tag{23}$$

where cross-terms do not contribute. We observe that, for general $\theta \neq \pi/4$, $i.e.$, $B_0 \neq 0$, as seen in (22), there is a difference between the populations of χ_1 and χ_2:

$$N_{\chi_1} - N_{\chi_2} = \cos 2\theta \Big(<n_{\psi^c}> - <n_\psi> \Big) \ , \tag{24}$$

where $<n_\psi> = <: \psi^\dagger \psi :> \neq <n_{\psi^c}> = <: \psi^{c\dagger} \psi^c :>$ are the corresponding number operators for the energy eigenstates.

This difference in the neutrino and antineutrino populations (15) is made possible by the presence of fermion-number-violating fermion-antifermion oscillations, whose probability was calculated in [26]:

$$\mathcal{P}(t) = |\langle \nu_1(t) | \nu_2(0) \rangle|^2 = \sin^2\theta \sin^2\Big(\frac{E_\nu - E_{\nu^c}}{2} t\Big) = \frac{m^2}{B_0^2 + m^2} \sin^2(B_0 t) \ , \tag{25}$$

where we used (7) with $\vec{B} = 0$, as in our specific background, and the definition of the mixing angle (22). The time evolution of the system is calculated using $|\psi\rangle$ and $|\psi^c\rangle$, which are eigenstates of the Hamiltonian.

In the case of relativistic neutrinos moving close to the speed of light, the oscillation length obtained from (25) is

$$L = \frac{\pi \hbar c}{|B_0|} = \frac{6.3 \times 10^{-14}\,\mathrm{GeV}}{B_0} \ \mathrm{cm}. \tag{26}$$

where we have reinstated $\hbar$ and c, and B_0 is measured in GeV. For oscillations to be effective at any given epoch in the early Universe, this length has to be less than the size of the Hubble horizon. We assume that a cosmological solution of the form discussed in [22], with a scale factor increasing linearly with time, is applicable some time after any earlier inflationary epoch. For a temperature $T_d \sim 10^9$ GeV, the relevant Hubble horizon size $\sim 10^{-12}$ cm. On the other hand, we see from (16) that the correct order of magnitude for the lepton asymmetry $\sim 10^{-10}$ is obtained if $B_0 \sim 10^{-1}$ GeV. For this value of B_0, the oscillation length (26) $\sim 10^{-13}$ cm, which is within the Hubble horizon size $\sim 10^{-12}$ cm. This implies that neutrino/antineutrino oscillations occur for sufficiently rapidly to establish chemical equilibrium and hence a lepton asymmetry [4].

In the above we considered a single generation of Majorana neutrino, without a Dirac mass term, m_D, which is required in seesaw mechanisms for the generation of light neutrino masses. The inclusion of a Dirac mass term modifies the mass matrix (18) via additional contributions to the diagonal mass terms that have the same sign for the two-component neutrino ψ and its conjugate spinor ψ^c:

$$\mathcal{M} = \begin{pmatrix} m_D - B_0 & -m \\ -m & m_D + B_0 \end{pmatrix} \ , \tag{27}$$

which modifies the mass eigenvalues (21) to

$$m_{1,2}^s = m_D \mp \sqrt{B_0^2 + m^2} \ . \tag{28}$$

In this case, following the above steps and setting $|\vec{p}| \simeq E$, the average energy of an ultra-relativistic neutrino, one can recalculate the oscillation probability and oscillation length. For ultra-relativistic neutrinos $p \gg m$, the oscillation probability becomes [26]

$$\mathcal{P}^s(t) \simeq \frac{m^2}{B_0^2 + m^2} \sin^2\Big(\frac{m_D \sqrt{B_0^2 + m^2}}{E} t\Big) \ , \tag{29}$$

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[4] Since the coupling of fermions to torsion is universal, the background would also couple to quarks and charged leptons. However, the splitting of a four-component spinor into a pair of two-component ones, and the subsequent derivation of the above results on oscillations between fermions and antifermions, is valid exclusively for neutrinos and cannot apply to quarks or charged leptons. Fermion-antifermion oscillations for these species would be forbidden by charge conservation.

leading to an energy-dependent oscillation length:

$$L^s \simeq \frac{\pi E}{m_D \sqrt{B_0^2 + m^2}} \; .\tag{30}$$

Flavour mixing in the presence of a background B_0 can also be considered, with effects on flavour oscillations that fall beyond the scope of our discussion here. We refer the reader to [26], where flavour oscillations in the presence of background B_0 have been studied in some detail.

III. OUTLOOK

As we have seen, the stringy cosmology proposed in [22] introduces the possibility of a constant torsion background in which a neutrino-antineutrino asymmetry develops as long as $B_0 \neq 0$. Clearly, more work is needed to flesh out the details of a possible leptogenesis scenario exploiting this CPTV mechanism, but we conclude by discussing some of the open issues.

It is clear that B_0 is negligible in the present-day Universe, which is characterized by a Minkowski space-time background. A phase transition directly from the value of $B_0 \neq 0$ during leptogenesis to $B_0 = 0$ is the simplest possibility, on the other hand in [22] a series of phase transitions was envisaged, in which the central charge deficit of the model Q^2 gradually reduced step-wise to zero. At each phase transition the value of the H-torsion background would change from a larger to a smaller value. In this approach, conformal field theories with different central charges characterize different epochs of the early Universe [22].

In the simplest model, in which there is a single phase transition at a temperature T_d at which B_0 switches off, this temperature would correspond to a decoupling temperature for the fermion-number-violating torsion-induced interactions in this model. The lepton number violation would freeze out, leaving a lepton asymmetry (16) that would be converted partially into a baryon asymmetry via sphaleron transitions in the Standard Model, as in other leptogenesis scenarios. In terms of the underlying neutrino-antineutrino oscillations, T_d would also be also the temperature at which such oscillations stop. We emphasize, however, that, unlike other leptogenesis scenarios, no CP violation is necessary. More complicated scenarios could also be envisaged, in which the non-zero torsion relaxes to zero in a series of discrete steps, as originally proposed in [22], but this is not essential for our purposes.

States with different values of B_0 correspond in the scenario of [22] to different conformal (fixed) points in moduli space. Transitions between such states correspond to phase transitions in the early universe that can be described by non-conformal (Liouville) time evolution, as discussed in [29]. A fuller study of the scenario for leptogenesis described here should include a discussion of such transitions, which lies beyond the scope of this exploratory work.

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